Locally bounded enriched categories

Jason Parker

(joint work with Rory Lucyshyn-Wright)

Brandon University, Manitoba, Canada

Australian Category Seminar
November 9, 2021[1]

Natural Sciences and Engineering Research Council of Canada

Conseil de recherches en sciences naturelles et en génie du Canada

Canada

[1] We acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC). Nous remercions le Conseil de recherches en sciences naturelles et en gnie du Canada (CRSNG) de son soutien.

Introduction

- Locally bounded ordinary categories were (implicitly) introduced by Freyd and Kelly in
  - [3] P.J. Freyd and G.M. Kelly. Categories of continuous functors I. *Journal of Pure and Applied Algebra* Vol. 2, Issue 3, 169-191, 1972.

  as a context for proving reflectivity results for orthogonal subcategories and categories of models.

- The notion of locally bounded (symmetric monoidal closed) category was then explicitly defined by Kelly in [5, Chapter 6] and used as the basis for a general treatment of enriched limit theories.

Introduction

- Locally bounded categories subsume locally presentable categories and many “topological” categories that are *not* locally presentable.

- Speaking of locally presentable categories, in

  - [4] G.M. Kelly. Structures defined by finite limits in the enriched context I. *Cahiers de Topologie et Géométrie Catégoriques Différentielle* 23, No. 1, 3-42, 1982.

  Kelly defined the notion of a locally presentable $\mathscr{V}$-category over a locally presentable closed category $\mathscr{V}$.

- Kelly *did* define the notion of a locally bounded closed category $\mathscr{V}$ in [5, Chapter 6], but never defined the notion of a locally bounded *$\mathscr{V}$-category* over such a $\mathscr{V}$. That's where this talk comes in!

Locally bounded (ordinary) categories

- Let's start by reviewing the definition of a locally bounded (ordinary) category. A **factegory** is a category $\mathscr{C}$ with a proper factorization system $(\mathscr{E}, \mathscr{M})$. The factegory $\mathscr{C}$ is **cocomplete** if $\mathscr{C}$ is cocomplete and has arbitrary cointersections (i.e. wide pushouts) of $\mathscr{E}$-morphisms.

- Given a small $\mathscr{M}$-sink $(m_i : C_i \to C)_{i \in I}$ in $\mathscr{C}$, its **union** can be defined as the $\mathscr{M}$-subobject m obtained from the $(\mathscr{E}, \mathscr{M})$-factorization

$$\coprod_i C_i \xrightarrow{e} \bigcup_i C_i \xrightarrow{m} C.$$

The sink $(m_i)_i$ is α-**filtered** if any sub-sink of size $< \alpha$ factors through some m_i.

Locally bounded (ordinary) categories

- A functor $U : \mathscr{C} \to \mathscr{D}$ between cocomplete factegories that preserves $\mathscr{M}$ is said to **preserve** (α-**filtered**) $\mathscr{M}$-**unions** if for any (α-filtered) $\mathscr{M}$-sink $(m_i)_i$ with union m, Um is the union of the $\mathscr{M}$-sink $(Um_i)_i$. If U preserves $\mathscr{M}$ and preserves α-filtered $\mathscr{M}$-unions, we also say that U is α-**bounded**.

- In particular, an object $C \in \mathbf{ob}\mathscr{C}$ of a cocomplete factegory $\mathscr{C}$ is α-bounded if $\mathscr{C}(C, -) : \mathscr{C} \to \mathbf{Set}$ preserves α-filtered $\mathscr{M}$-unions.

- Finally, a set $\mathscr{G} \subseteq \mathbf{ob}\mathscr{C}$ of a cocomplete factegory is an $(\mathscr{E}, \mathscr{M})$-**generator** if for any $C \in \mathbf{ob}\mathscr{C}$, the canonical morphism

$$\coprod_{G \in \mathscr{G}} \mathscr{C}(G, C) \cdot G \to C$$

lies in $\mathscr{E}$ (equivalently, the functors $\mathscr{C}(G, -) : \mathscr{C} \to \mathbf{Set}$ $(G \in \mathscr{G})$ are **jointly** $\mathscr{M}$-**conservative** [6]).

Locally bounded (ordinary) categories

Definition (Kelly [5])

A locally α-bounded category is a cocomplete factegory $\mathscr{C}$ with an $(\mathscr{E}, \mathscr{M})$-generator consisting of α-bounded objects.

Note the parallel with locally α-presentable categories: a locally α-presentable category is a cocomplete category $\mathscr{C}$ with a *strong* generator consisting of α-*presentable* objects.

Examples

- Any locally α-presentable category [3, 3.2.3], with $(\mathscr{E}, \mathscr{M}) = (\mathbf{StrongEpi}, \mathbf{Mono})$ and the given strong generator of α-presentable (and hence α-bounded) objects.

- Any topological category over **Set** is locally $\aleph_0$-bounded [11, 2.3], with $(\mathscr{E}, \mathscr{M}) = (\mathbf{Epi}, \mathbf{StrongMono})$ and the generator consisting of just the discrete object on $\{*\}$.

- Any cocomplete locally cartesian closed category with a generator and arbitrary cointersections of epimorphisms, so that $(\mathscr{E}, \mathscr{M}) = (\mathbf{Epi}, \mathbf{StrongMono})$. These include the concrete quasitoposes of Dubuc [2].

Locally bounded closed categories

We now recall Kelly's definition of locally bounded symmetric monoidal closed category:

Definition (Kelly [5])

A symmetric monoidal closed category $\mathscr{V}$ is **locally α-bounded as a closed category** if $\mathscr{V}_0$ is locally α-bounded, the proper factorization system $(\mathscr{E}, \mathscr{M})$ is enriched, the unit object $I \in \mathbf{ob}\mathscr{V}$ is α-bounded, and $G \otimes G'$ is α-bounded for all $G, G' \in \mathscr{G}$.

For example: any symmetric monoidal closed category $\mathscr{V}$ with $\mathscr{V}_0$ locally α-presentable [5, Chapter 6]; any commutative unital quantale; any (cartesian closed) topological category over **Set**; any cocomplete locally cartesian closed category with generator and arbitrary epi-cointersections (e.g. any concrete quasitopos).

$\mathscr{V}$-factegories

- For the remainder of the talk, $\mathscr{V}$ will be a locally α-bounded closed category (sometimes a stronger assumption than needed).
- An enriched factorization system $(\mathscr{E}_\mathscr{C}, \mathscr{M}_\mathscr{C})$ on a $\mathscr{V}$-category $\mathscr{C}$ [7] is **compatible** with $(\mathscr{E}, \mathscr{M})$ if each $\mathscr{C}(C, -) : \mathscr{C} \to \mathscr{V}$ $(C \in \mathbf{ob}\mathscr{C})$ preserves the right class.
- A $\mathscr{V}$-**factegory** is a $\mathscr{V}$-category $\mathscr{C}$ with an enriched proper factorization system $(\mathscr{E}_\mathscr{C}, \mathscr{M}_\mathscr{C})$ that is compatible with $(\mathscr{E}, \mathscr{M})$. The $\mathscr{V}$-factegory $\mathscr{C}$ is **cocomplete** if the $\mathscr{V}$-category $\mathscr{C}$ is cocomplete and has arbitrary (conical) cointersections of $\mathscr{E}$-morphisms.

Enriched $(\mathscr{E}, \mathscr{M})$-generators

- Let $\mathscr{C}$ be a cocomplete $\mathscr{V}$-factegory. A set $\mathscr{G} \subseteq \mathbf{ob}\mathscr{C}$ is an **enriched $(\mathscr{E}, \mathscr{M})$-generator** if for each $C \in \mathbf{ob}\mathscr{C}$, the canonical morphism $\coprod_{G \in \mathscr{G}} \mathscr{C}(G, C) \otimes G \to C$ lies in $\mathscr{E}$.

- A set $\mathscr{G} \subseteq \mathbf{ob}\mathscr{C}$ is an enriched $(\mathscr{E}, \mathscr{M})$-generator iff the representable $\mathscr{V}$-functors $\mathscr{C}(G, -) : \mathscr{C} \to \mathscr{V}$ $(G \in \mathscr{G})$ are **jointly $\mathscr{M}$-conservative**.

Enriched α-bounded objects

Let $\mathscr{C}$ be a cocomplete $\mathscr{V}$-factegory. An object $C \in \mathbf{ob}\mathscr{C}$ is an **enriched α-bounded object** if $\mathscr{C}(C, -) : \mathscr{C} \to \mathscr{V}$ preserves α-filtered $\mathscr{M}$-unions.

Definition

A **locally α-bounded** $\mathscr{V}$-category is a cocomplete $\mathscr{V}$-factegory $\mathscr{C}$ with an enriched $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{G}$ consisting of enriched α-bounded objects.

- Note the parallel with locally α-presentable $\mathscr{V}$-categories: a locally α-presentable $\mathscr{V}$-category is a cocomplete $\mathscr{V}$-category with an enriched *strong* generator of enriched *α-presentable* objects [4, 3.1].
- If $\mathscr{V}$ is a locally α-bounded closed category with ordinary $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{G}$, then $\mathscr{V}$ is itself a locally α-bounded $\mathscr{V}$-category with enriched $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{G}$.
- Any locally bounded $\mathscr{V}$-category is total and complete.

Bounding right adjoints

The following notion of *bounding right adjoint* has proved to be fundamental for constructing examples of locally bounded $\mathscr{V}$-categories:

Definition

Let $U : \mathscr{C} \to \mathscr{D}$ be a $\mathscr{V}$-functor between cocomplete $\mathscr{V}$-factegories. Then U is an α-**bounding right adjoint** if U is α-bounded and has a left adjoint whose counit is pointwise in $\mathscr{E}$.

U is an α-bounding right adjoint iff U is α-bounded, has a left adjoint, and is $\mathscr{M}$-conservative, iff U is α-bounded, has a left adjoint, and reflects $\mathscr{E}$. An α-bounding right adjoint is automatically ($\mathscr{V}$-)faithful.

Bounding right adjoints

Theorem

Let $\mathscr{C}$ be a cocomplete $\mathscr{V}$-factegory and let $\mathscr{G} \subseteq \mathbf{ob}\mathscr{C}$ be a set. Then $\mathscr{C}$ is locally α-bounded with enriched $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{G}$ iff the nerve $\mathbf{y}_{\mathscr{G}} : \mathscr{C} \to [\mathscr{G}^{\mathbf{op}}, \mathscr{V}]$ is an α-bounding right adjoint.

Theorem

Let $\mathscr{D}$ be a locally α-bounded $\mathscr{V}$-category and $\mathscr{C}$ a cocomplete $\mathscr{V}$-factegory. If $U : \mathscr{C} \to \mathscr{D}$ is an α-bounding right adjoint, then $\mathscr{C}$ is locally α-bounded.

Bounding right adjoints

Theorem

Let $\mathscr{C}$ be a cocomplete $\mathscr{V}$-factegory. Then $\mathscr{C}$ is locally α-bounded iff there exists a small $\mathscr{V}$-category $\mathscr{A}$ and an α-bounding right adjoint $U : \mathscr{C} \to [\mathscr{A}, \mathscr{V}]$, i.e. a $\mathscr{V}$-functor $U : \mathscr{C} \to [\mathscr{A}, \mathscr{V}]$ that is α-bounded, $\mathscr{M}$-conservative, and has a left adjoint.

Note the parallel with Kelly's result [5, 3.1]: a cocomplete $\mathscr{V}$-category $\mathscr{C}$ is locally α-presentable iff there exists a small $\mathscr{V}$-category $\mathscr{A}$ and a $\mathscr{V}$-functor $U : \mathscr{C} \to [\mathscr{A}, \mathscr{V}]$ that has rank α, is conservative, and has a left adjoint.

Corollary: if $\mathscr{C}$ is locally α-bounded and $\mathscr{A}$ is small, then $[\mathscr{A}, \mathscr{C}]$ is locally α-bounded.

Enriched vs. ordinary local boundedness

Recall that $\mathscr{V}$ is a locally α-bounded closed category with ordinary $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{G}$.

Theorem

If $\mathscr{C}$ is locally α-bounded with enriched $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{H}$, then $\mathscr{C}_0$ is locally α-bounded with ordinary $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{G} \otimes \mathscr{H}$.

Theorem

If $\mathscr{C}$ is a cocomplete $\mathscr{V}$-factegory such that $\mathscr{C}_0$ is locally bounded with ordinary $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{H}$, then $\mathscr{C}$ is locally bounded with enriched $(\mathscr{E}, \mathscr{M})$-generator $\mathscr{H}$.

A representability theorem

It is well known that if $\mathscr{C}$ is a locally presentable (even accessible) category, then a functor $U : \mathscr{C} \to \mathbf{Set}$ is *representable* iff U is continuous and has rank. We have a similar result for locally bounded categories:

Theorem

Let $\mathscr{C}$ be a locally bounded and $\mathscr{E}$-cowellpowered $\mathscr{V}$-category. If $U : \mathscr{C} \to \mathscr{V}$ preserves $\mathscr{M}$, then U is representable iff U is continuous and bounded.

Adjoint functor theorems

Recall that a functor $U : \mathscr{C} \to \mathscr{D}$ between locally presentable categories has a left adjoint iff U is continuous and has rank.

Theorem

Let $\mathscr{C}, \mathscr{D}$ be locally bounded $\mathscr{V}$-categories such that $\mathscr{C}$ is $\mathscr{E}$-cowellpowered. If $U : \mathscr{C} \to \mathscr{D}$ preserves $\mathscr{M}$, then U has a left adjoint iff U is continuous and bounded.

Recall that if $\mathscr{C}$ is locally presentable and $\mathscr{D}$ arbitrary, then $F : \mathscr{C} \to \mathscr{D}$ has a right adjoint iff F is cocontinuous.

Theorem (cf. [6])

Let $F : \mathscr{C} \to \mathscr{D}$ be a $\mathscr{V}$-functor from a locally bounded $\mathscr{V}$-category $\mathscr{C}$ to an arbitrary $\mathscr{V}$-category $\mathscr{D}$. Then F has a right adjoint iff F is cocontinuous.

(In fact, $\mathscr{C}$ just needs to be cocomplete $\mathscr{V}$-factegory with enriched $(\mathscr{E}, \mathscr{M})$-generator.)

α-bounded-small limits

- It is well known that α-small limits commute with α-filtered colimits in any locally α-presentable category.
- If $\mathscr{V}$ is a locally α-presentable closed category, then Kelly defined in [4, 4.1] the notion of an *α-small weight* $W : \mathscr{B} \to \mathscr{V}$: $|\mathbf{ob}\mathscr{B}| < \alpha$, $\mathscr{B}(B, B') \in \mathscr{V}_\alpha$ for all $B, B' \in \mathbf{ob}\mathscr{B}$, and $WB \in \mathscr{V}_\alpha$ for all $B \in \mathbf{ob}\mathscr{B}$.
- He then showed in [4, 4.9] that α-small weighted limits commute with conical α-filtered colimits in any locally α-presentable $\mathscr{V}$-category.
- If $\mathscr{V}$ is a locally α-bounded closed category, we can define the similar notion of an *α-bounded-small* weight $W : \mathscr{B} \to \mathscr{V}$.

α-bounded-small limits

Definition

Let $\mathscr{V}$ be a locally α-bounded closed category. A weight $W : \mathscr{B} \to \mathscr{V}$ is α-**bounded-small** if $|\mathbf{ob}\mathscr{B}| < \alpha$, $\mathscr{B}(B, B')$ is an enriched α-bounded object of $\mathscr{V}$ for all $B, B' \in \mathbf{ob}\mathscr{B}$, and WB is an enriched α-bounded object of $\mathscr{V}$ for all $B \in \mathbf{ob}\mathscr{B}$.

Kelly showed in [4, 4.3] that the saturation of the class of α-small weights is equal to the saturation of the class of weights for α-small conical limits and α-presentable cotensors. We similarly have:

Theorem

The saturation of the class of α-bounded-small weights is equal to the saturation of the class of weights for α-small conical limits and α-bounded cotensors.

α-bounded-small limits

Definition

Let $\mathscr{C}$ be a complete and cocomplete $\mathscr{V}$-factegory and $W : \mathscr{B} \to \mathscr{V}$ a small weight. Then **W-limits commute with α-filtered $\mathscr{M}$-unions in $\mathscr{C}$** if the W-limit $\mathscr{V}$-functor $\{W, -\} : [\mathscr{B}, \mathscr{C}] \to \mathscr{C}$ is α-bounded.

Theorem

If $\mathscr{C}$ is a locally α-bounded $\mathscr{V}$-category, then α-bounded-small weighted limits commute with α-filtered $\mathscr{M}$-unions in $\mathscr{C}$.

Reflectivity and local boundedness

- Freyd and Kelly proved in [3, 4.1.3, 4.2.2] that if $\mathscr{C}$ is an $\mathscr{E}$-cowellpowered locally bounded ordinary category and Θ is a "quasi-small" class of morphisms in $\mathscr{C}$, then the orthogonal subcategory $\Theta^{\perp} \hookrightarrow \mathscr{C}$ is reflective and locally bounded.
- Kelly showed in [5, Chapter 6] that the reflectivity still holds even without $\mathscr{E}$-cowellpoweredness.

We have enriched both results as follows:

Theorem

Let $\mathscr{C}$ be a locally bounded $\mathscr{V}$-category with a "quasi-small" class of morphisms Θ. Then the enriched orthogonal sub-$\mathscr{V}$-category $\Theta^{\perp_{\mathscr{V}}} \hookrightarrow \mathscr{C}$ is reflective, and $\Theta^{\perp_{\mathscr{V}}}$ is locally bounded if $\mathscr{C}$ is $\mathscr{E}$-cowellpowered.

Reflectivity and local boundedness

Freyd and Kelly also proved in [3, 5.2.1, 5.2.2] that if $\mathscr{C}$ is a locally bounded and $\mathscr{E}$-cowellpowered ordinary category and $(\mathscr{A}, \Phi)$ is a limit sketch, then Φ-**Mod**$(\mathscr{A}, \mathscr{C})$ is reflective in $[\mathscr{A}, \mathscr{C}]$ and locally bounded.

Theorem

*Let $\mathscr{C}$ be a locally α-bounded $\mathscr{V}$-category and $(\mathscr{A}, \Phi)$ an enriched limit sketch [5, 6.3]. Then the full sub-$\mathscr{V}$-category Φ-**Mod**$(\mathscr{A}, \mathscr{C}) \hookrightarrow [\mathscr{A}, \mathscr{C}]$ is reflective, and Φ-**Mod**$(\mathscr{A}, \mathscr{C})$ is also locally bounded if $\mathscr{C}$ is $\mathscr{E}$-cowellpowered. If every weight in Φ is α-bounded-small, then Φ-**Mod**$(\mathscr{A}, \mathscr{C})$ is in fact locally α-**bounded**.*

*In particular, if $\mathscr{T}$ is a Φ-theory for a class of small weights Φ, then Φ-**Cts**$(\mathscr{T}, \mathscr{C})$ is reflective in $[\mathscr{T}, \mathscr{C}]$, and is locally bounded if $\mathscr{C}$ is $\mathscr{E}$-cowellpowered.*

Reflectivity and local boundedness

As a corollary, we obtain the following result for the enriched algebraic theories of Lucyshyn-Wright [8]:

Theorem

Let $\mathscr{J} \hookrightarrow \mathscr{V}$ be a small system of arities, let $\mathscr{T}$ be a $\mathscr{J}$-theory, and let $\mathscr{C}$ be a locally bounded and $\mathscr{E}$-cowellpowered $\mathscr{V}$-category. Then the full sub-$\mathscr{V}$-category $\mathscr{T}$-$\mathbf{Alg}(\mathscr{C}) \hookrightarrow [\mathscr{T}, \mathscr{C}]$ of the $\mathscr{T}$-algebras is reflective and locally bounded, and the forgetful $\mathscr{V}$-functor $U^{\mathscr{T}} : \mathscr{T}$-$\mathbf{Alg}(\mathscr{C}) \to \mathscr{C}$ is monadic.

In particular, if $\mathscr{C}$ is a locally α-bounded and $\mathscr{E}$-cowellpowered ordinary category and $\mathscr{T}$ is a Lawvere theory, then the category $\mathscr{T}$-$\mathbf{Alg}(\mathscr{C})$ of $\mathscr{T}$-algebras in $\mathscr{C}$ is reflective in $[\mathscr{T}, \mathscr{C}]$ and locally α-bounded, and the forgetful functor $U^{\mathscr{T}} : \mathscr{T}$-$\mathbf{Alg}(\mathscr{C}) \to \mathscr{C}$ is monadic.

In summary...

- We have defined a notion of locally bounded $\mathscr{V}$-category over a locally bounded closed category $\mathscr{V}$, which enriches the locally bounded ordinary categories of Freyd and Kelly, and parallels Kelly's notion of locally presentable $\mathscr{V}$-category over a locally presentable closed category $\mathscr{V}$.
- Examples of locally bounded closed categories include locally presentable closed categories, commutative unital quantales, topological categories over **Set**, and epi-cocomplete quasitoposes with generators.
- Many of the results for locally presentable enriched categories have analogues for locally bounded enriched categories: representability theorems, adjoint functor theorems, and commutation of suitably small limits with suitably filtered colimits/unions.

In summary...

- Moreover, locally bounded enriched categories admit full enrichments of Freyd and Kelly's reflectivity results for orthogonal subcategories and categories of models.
- Lucyshyn-Wright and I have also shown that locally bounded enriched categories provide a fruitful setting for obtaining results on free monads, presentations of monads, and algebraic colimits of monads for a subcategory of arities (to be presented in forthcoming work [10]).
- The content of this talk (and more!) is contained in the preprint:
    - [9] Rory B.B. Lucyshyn-Wright and Jason Parker, *Locally bounded enriched categories*, Preprint, arXiv:2110.07072, 2021.

α-bounded monads?

- One topic I did *not* touch on is the local boundedness of Eilenberg-Moore categories. It is (well) known that if $\mathscr{C}$ is a locally α-presentable $\mathscr{V}$-category and $\mathbb{T}$ is a $\mathscr{V}$-monad on $\mathscr{C}$ with rank α, then the Eilenberg-Moore $\mathscr{V}$-category $\mathbb{T}$-**Alg** is locally α-presentable, and $U^{\mathbb{T}} : \mathbb{T}\text{-}\mathbf{Alg} \to \mathscr{C}$ is continuous and has rank α (see [1, 6.9]).

- Does an analogous result hold for α-bounded $\mathscr{V}$-monads on locally α-bounded $\mathscr{V}$-categories? Essentially yes, but with some slight subtleties/complications (to be presented in forthcoming work).

Thank you!

E-mail: parkerj@brandonu.ca

Website: www.jasonparkermath.com

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